VAPORIZATION IN THE PULSE HEATING OF LIQUIDS

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The problem of the boiling of a pulse-heated liquid is considered with allowance for spontaneous vapor-phase centers. The theory is compared with the experimental data.

Usually, in calculating vaporization mechanisms the contribution of fluctuation centers is not taken into account. However, as experiments on the pulse heating of liquids show [1, 2], under certain conditions vaporization depends significantly on spontaneous centers of the fluctuation type. These centers develop at a sufficiently high degree of superheating, and their number may become so great that the bubbles growing at them make the chief contribution to vapor formation while the artificial centers play a secondary role. There have also been reports of liquids vaporized in a laser beam [3]. The heating time in experiments [1–3] was 10^{-3}–10^{4} sec. A similar situation develops when sufficiently high tensile stresses are suddenly created in a liquid causing cavitation.

The frequency of fluctuational nucleation J, $\mathrm{cm}^{-3} \cdot \mathrm{sec}^{-1}$, was theoretically determined in [4–6]:

$$J \sim \exp\left[-W/kT'\right]. \tag{1}$$

According to Zel'dovich's estimate [5] the time required to reach a stationary distribution of embryonic bubbles with respect to size is very short (about 10^{-10} sec); therefore nonstationarity effects will be neglected. In the subsequent calculations the function J is given in the form

$$J = B\exp\Phi, \tag{2}$$

where Φ depends on the pressure and temperature of the liquid and B = const. The relation between the rates of growth of the vapor bubble volume and time is taken in the form

$$\dot{v}(\tau) = kb(\theta)\,\tau^{k-1}. \tag{3}$$

If the growth of the bubble is restrained by inertia forces at its boundary (the Rayleigh case), then k = 3, whereas if the rate of growth is limited by the supply of heat, then $k \simeq 1/2$. The coefficient b depends on the superheating of the liquid. We note that in the pulse regime there is no need to take into account convection currents in the liquid at least in the initial stage of vaporization.

We will establish the criterion of impulsive heating of a boiling liquid when an important part is played by spontaneous centers. The fraction of the liquid converted into vapor β at moment τ is given by the integral equation

$$\beta(\tau) = \frac{\rho''}{\rho'}\int_0^\tau J(T')\left[1-\beta(t)\right]\times$$
$$\times\left\{\int_0^{\tau-t} kb(t+\zeta)\,\zeta^{k-1}\,d\zeta\right\}dt +$$
$$+\frac{\rho''}{\rho'}\Omega\int_0^\tau \dot{v}(t,\ \theta)\,dt. \tag{4}$$

The time τ is reckoned from the moment at which $\theta = T' - T_S = 0$. The temperature of the liquid T' depends on the method of heating. The first integral on the right side of the equation describes the vapor formation at spontaneous centers.

Let the heat q be released in the liquid per unit initial volume per unit time. We write the heat balance equation in the adiabatic approximation

$$\int_0^\tau \frac{q}{\rho'}\,d\tau = \theta\,\overline{c'}(1-\beta) + \beta L. \tag{5}$$

The temperature of the vapor is taken equal to T_S. Without account for fluctuational nucleation the solution of system (4), (5) has the form

$$\frac{\overline{c'}}{L}\theta(\tau) = \left(\frac{1}{L}\int_0^\tau \frac{q}{\rho'}\,d\tau - \frac{\rho''}{\rho'}\Omega\int_0^\tau \dot{v}\,d\tau\right)\times$$
$$\times\left(1-\frac{\rho''}{\rho'}\Omega\int_0^\tau \dot{v}\,d\tau\right)^{-1}. \tag{6}$$

An analysis of this equation at q/ρ' = const shows that for small q the temperature of the liquid at first rises, but with the development of vapor formation at the existing centers passes through a maximum and tends to T_S. At large q the temperature increases monotonically. An estimate based on the theory of [4–6] shows that spontaneous nucleation occurs at a temperature $\theta \leqslant L/\bar{c}'$. It is convenient to introduce a pulse regime criterion in terms of the dimensionless complex

$$K = \frac{q}{L\rho'}\left(\frac{\rho''}{\rho'}\Omega\bar{b}\right)^{-1/k}. \tag{7}$$

The inequality K > 1 serves as the condition of attainment of the intense nucleation temperature. According to Eq. (6) it corresponds to an infinite increase in the temperature of the liquid. At K ≫ 1 fluctuational nucleation occurs even at small β, and the artificial centers play almost no role.

If the heating of the liquid is so organized that $T' = \text{const}$, then from Eq. (4) we can easily obtain a direct estimate of the vapor fraction of fluctuational origin ε:

$$\varepsilon(1-\varepsilon)^{-\frac{k+1}{k}} \simeq \frac{J(T')}{\Omega(k+1)}\left(\frac{\rho'}{\rho''}\,\frac{\beta}{\Omega b}\right)^{1/k}. \tag{8}$$

We shall now consider Eq. (4) when $\beta \ll 1$. We represent the variation of liquid temperature with time by a power function

$$\theta(\tau) \sim \tau^{n}. \tag{9}$$

Moreover, we assume that $K \gg 1$. Comparison of Eq. (1) with the expression (2) and the explicit form of the relation $W(T')$ shows that

$$\theta\frac{d\Phi}{dT} \gg 1; \quad \frac{d^2\Phi}{dT^2}\left(\frac{d\Phi}{dT}\right)^{-2} \ll 1. \tag{10}$$

Then the integrand in Eq. (4) describing vapor formation at fluctuation centers has a sharply expressed maximum at $t = t_m$, and $\tau - t_m \simeq k[\dot{\Phi}(\tau)]^{-1} \ll \tau$. In the region of the maximum it is sufficient to confine oneself to the linear term in the expansion of $\Phi(t)$ in the neighborhood of the point τ. In these approximations expression (4) is simplified and may be represented in the following form:

$$v''(\tau) \simeq \Gamma(k+1)[\dot{\Phi}(\tau)]^{-k-1} b(\tau) J(\tau). \tag{11}$$

We now turn to the solution of the problem of a wire immersed in a test liquid and heated by a pulse of electric current. As shown in [1], the corresponding differential equation can be written in the form of a one-dimensional heat conduction equation with homogeneous boundary and initial conditions

$$c'\rho'\frac{\partial\theta}{\partial\tau} = \frac{\partial}{\partial x}\lambda'\frac{\partial\theta}{\partial x} + q\left(x,\ \tau,\ \theta,\ \left.\frac{\partial\theta}{\partial\tau}\right|_{x=0}\right),$$

$$\theta(x=\infty,\ \tau) = \theta(x,\ \tau=0) = 0. \tag{12}$$

The function q describes the heat sources and sinks:

$$q = 2\omega\delta(x) - \frac{\rho cd}{2}\left.\frac{\partial\theta}{\partial\tau}\right|_{x=0}\delta(x) - L\rho''\dot{v}''.$$

It is assumed that the wire material has good wettability with respect to the liquid.

Differentiating (11) with respect to time, we obtain

$$\dot{v}''(\tau) \simeq \Gamma(k+1)[\dot{\Phi}(\tau)]^{-k} b(\tau) J(\tau). \tag{13}$$

Here and henceforth the approximations are based on inequalities (10). We represent the solution of Eq. (12) in the form $\theta = \theta_+ + \theta_-$, θ_+ being the solution of the unperturbed problem ($\dot{v}'' = 0$). Then for the perturbation θ_- we obtain the equation

$$c'\rho'\frac{\partial\theta_-}{\partial\tau} = \frac{\partial}{\partial x}\lambda'\frac{\partial\theta_-}{\partial x} -$$

$$-\rho c\frac{d}{2}\delta(x)\left.\frac{d\theta_-}{d\tau}\right|_{x=0} - \rho''L\dot{v}'',$$

$$\theta_-(x,\ 0) = \theta_-(\infty,\ \tau) = 0. \tag{14}$$

We assume that in the unperturbed case the temperature of the wire $\theta_+(0, \tau)$ varies according to the law (9). Then the solution of the unperturbed problem coincides with the known solution of the problem of the temperature field near a wall [7]:

$$\theta_+(x,\ \tau) = \theta_+(0,\ \tau)\,2^{2n}\,\Gamma(n+1)\times$$

$$\times\frac{2}{\sqrt{\pi}}\int^{\infty}\frac{(\zeta-\gamma)^{2n}}{(2n)!}\exp[-\zeta^2]\,d\zeta, \quad \gamma = \frac{x}{2}\sqrt{\rho'c'/\lambda'\tau}. \tag{15}$$

For the approximate solution of problem (12) it is sufficient to describe the temperature field correctly at

$$\gamma = \frac{x}{2}\sqrt{\rho'c'/\lambda'\tau} \ll 1. \tag{16}$$

This latter inequality means that we must consider a thin wall layer x forming only a small part of the heated layer $x_0 = 2(3\lambda'\tau/\rho'c')^{1/2}$ [8]. In this case, neglecting terms quadratic in γ, instead of (15) we obtain

$$\theta_+(x,\ \tau) \simeq \theta_+(0,\ \tau)\left[1 - \frac{\Gamma(n+1)}{\Gamma(n+1/2)}\,x\sqrt{\frac{\rho'c'}{\lambda'\tau}}\right]. \tag{17}$$

Equation (14) reduces to the integral equation [9]

$$\theta_-(x,\ \tau) + \int_0^{\tau}\frac{1}{\rho'c'}\times$$

$$\times\int_{-\infty}^{+\infty}\left[\frac{\rho cd}{2}\,\delta(\zeta)\left.\frac{d\theta_-}{d\tau}\right|_{x=0} + \rho''L\dot{v}''\right]\times$$

$$\times\frac{\exp[-(\zeta-x)^2\rho'c'/4\lambda'(\tau-t)]}{2\sqrt{\lambda'\pi(\tau-t)}}\sqrt{\rho'c'}\,d\zeta\,dt = 0.$$

In the approximations already employed the solution of this equation has the form

$$-\theta_-(x,\ \tau) = \frac{GJ(0,\ \tau)}{\left|\dfrac{\partial\Phi}{\partial x}\right|_{x=0}\left(1 + R\sqrt{\dot{\Phi}(0,\ \tau)}\right)}\times$$

$$\times\exp\left[-x\sqrt{\frac{\dot{\Phi}(0,\ \tau)\rho'c'}{\lambda'}}\right], \tag{18}$$

where

$$R = \frac{\rho cd}{4\sqrt{\lambda'\rho'c'}}; \quad G = \frac{\rho''Lb(\tau)\Gamma(k+1)}{\sqrt{\lambda'\rho'c'}\left[\dot{\Phi}(0,\ \tau)\right]^{k+1/2}}.$$

In the experiments on pulse-heated wires in a liquid [1, 2], $R(\dot{\Phi})^{1/2} \gg 1$. In this case formula (18) takes the simpler form

$$-\theta_-(0,\ \tau) \simeq \frac{4\rho''L}{\rho cd}\,\frac{b(\tau)\Gamma(k+1)}{\left[\dot{\Phi}(0,\ \tau)\right]^{k+1}}\,\frac{J(0,\ \tau)}{\left|\dfrac{\partial\Phi}{\partial x}\right|_{x=0}}. \tag{19}$$

Experimentally, a sharp perturbation due to vigorous boiling is observed in the monotonic variation of the wire temperature θ_+ with time. It is possible to estimate the observed value of the perturbation θ_- and measure the temperature T^* at which it occurs. In the experiments [1, 2] $\theta_- \sim 10^{-1}$–10^{-2}°C. The temperature T^* can be calculated using formulas (1) and (19) and compared with the experimentally measured temperature. Such a comparison has been made for n-hexane and methyl alcohol at various pressures P. In both cases good agreement is obtained between the experimental and calculated values of T*. This shows that vapor formation at fluctuation centers actually has considerable importance in connection with the rapid transfer of the liquid to the metastable state. The data for n-hexane are presented in the table.

Data on the Pulse Heating of n-Hexane ($\tau \sim 3 \cdot 10^{-4}$ sec)

P, bar	T_s, °C	T*, °C	
		Experiment	Calculation
1.0	68.7	189	188
2.9	106	192	191
5.0	130	195	194
11.2	172	205	203
16.0	193	213	211
20.6	209	220	218
25.7	224	228	227

From formula (19) it can be seen that in principle it is possible to use experiments with different pulse durations to study the temperature behavior of the spontaneous nucleation frequency.

NOTATION

$J = B \exp \Phi$ is the nucleation frequency; W is the work of formation of critical nucleus; x, d are the distance from the surface and the diameter of the wire; q is the power of heat source per unit volume of liquid; β is the fraction of initial mass of liquid converted into vapor; Ω is the density of artificial vapor phase centers; v is the volume of vapor bubble ($v = b\tau^k$); v" is the volume of vapor per unit volume of the liquid; τ is the time; T is the temperature; $\theta = T' - T_s$; θ_+ and θ_- are the unperturbed temperature and perturbation due to boiling; k is Boltzmann's constant or bubble growth exponent; L is the latent heat of evaporation; ρ is the density; c is the specific heat; λ is the thermal conductivity; $\Gamma(k)$ and $\delta(t)$ are the gamma and delta functions; ω is the rate of heat release in wire per unit area of lateral surface. The quantities λ', ρ', c', T' relate to the liquid; the quantities λ, ρ, c to the wire; the quantities ρ'', T" to the vapor. A dot denotes differentiation with respect to time; the subscript s denotes the saturation line. A prime denotes the mean value.

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